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M. RASHID AND N. ALAM

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EXACT TRAVELING WAVE SOLUTIONS FOR THE (2+1)-DIMENSIONAL BURGERS EQUATION VIA THE NEW APPROACH OF GENERALIZED (G'/G)-EXPANSION METHOD

M. RASHID[1*] AND N. ALAM[2]

[1]Department of Mathematics, Hajee Mohammad Danesh Science and Technology University, Dinajpur-5200; [2]Department of Mathematics, Pabna University of Science and Technology, Pabna-6600, Bangladesh.

*Corresponding author & address: Mamunur Rashid, E-mail: mamun_hstu@yahoo.com
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ABSTRACT

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In this paper, the new approach of generalized (G'/G)-expansion method is applied to construct traveling wave solutions of the nonlinear evolution equation via the (2+1)-dimensional Burgers equation. This method is one of the powerful methods that appear in recent time for establishing exact traveling wave solutions of nonlinear partial differential equations. By this method we have obtained many new types of complexiton soliton solutions, such as, various combinations of trigonometric periodic function and rational function solutions, various combination of hyperbolic function and rational function solutions.

Key words: *The new generalized* (G'/G)*-expansion method, complexiton soliton, the (2+1)-dimensional Burgers equation, traveling wave solutions*
Mathematics Subject Classification: 35K99, 35P05, 35P99

INTRODUCTION

Nonlinear evolution equations have a major role in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations (Ablowitz and Clarkson, 1991; Alam and Akber, 2013). In recent years, quite a few methods for obtaining explicit traveling and solitary wave solutions of nonlinear evolution equations have been proposed (Zayed *et al.* 2004; Zayed *et al.* 2006). A variety of powerful methods, such as the Hirota's bilinear method (Hirota 2004), the rank analysis method (Feng 2000), the ansatz method (Hu 2001), the homotopy perturbation method (Mohyud-Din, 2007; Mohyud-Din and Noor, 2009), the exp-functions method (He and Wu, 2006), the modified simple equation method (Jawad *et al.* 2010), the Jacobi elliptic function expansion method (Liu 2005; Chen and Wang, 2005), the Adomian decomposition method (Adomain 1994), the homogeneous balance method (Wang 1995; Wang 1996), the F-expansion method (Wang and Li, 2005), the Backlund transformation method (Miura 1978), the Darboux transformation method (Matveev and Salle, 1991), $\exp(-\varphi(\xi))$-expansion method (Alam *et al.* 2014a; Alam *et al.* 2015a), the auxiliary equation method (Sirendaoreji 2007), the inverse scattering transform (Ablowitz and Clarkson, 19991), the complex hyperbolic function method (Chow 1995), the (G'/G)-expansion method (Wang *et al.* 2008), the novel (G'/G)-expansion method (Alam *et al.* 2014b; Alam *et al.* 2015b), the new generalized (G'/G)-expansion method (Naher and Abdullah, 2013; Zhang *et al.* 2008; Zhang *et al.* 2010) and so on.

The objective of this article is to apply the new generalized (G'/G)-expansion method to construct the exact solutions for nonlinear evolution equations in mathematical physics via the (2+1)-dimensional Burgers equations. This study shows solution with free parameters that might be important to explain some complex physical phenomena. This study also discussed the new generalized (G'/G)-expansion method that is quite efficient and practically well suited to be used in finding exact solutions.

MATERALS AND METHODS

The present work is a theoretical study where new generalized (G'/G)-expansion method has been applied for finding the exact solution.

Description of the new generalized (G'/G)-expansion method

Let us consider a general nonlinear PDE in the form

$$P(u,u_t,u_x,u_{tt},u_{tx},u_{xx},\cdots)=0, \tag{1}$$

where $u=u(x,t)$ is an unknown function, P is a polynomial in $u(x,t)$ and its derivatives in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives.

Step 1: We combine the real variables x and t by a complex variable ξ

$$u(x,t)=u(\xi),\ \xi=x\pm V\,t, \tag{2}$$

where V is the speed of the traveling wave. The traveling wave transformation (2) converts Eq. (1) into an ordinary differential equation (ODE) for $u = u(\xi)$:

$$Q(u, u', u'', u''', \cdots) = 0, \tag{3}$$

where Q is a polynomial of u and its derivatives and the superscripts indicate the ordinary derivatives with respect to ξ.

Step 2: According to possibility Eq. (3) can be integrated term by term one or more times, yields constant(s) of integration. The integral constant may be zero, for simplicity.

Step 3. Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$u(\xi) = \sum_{i=0}^{N} a_i (d + H)^i + \sum_{i=1}^{N} b_i (d + H)^{-i}, \tag{4}$$

where either a_N or b_N may be zero, but both a_N and b_N could be zero at a time, $a_i \ (i = 0, 1, 2, \cdots, N)$ and $b_i \ (i = 1, 2, \cdots, N)$ and d are arbitrary constants to be determined later and $H(\xi)$ is

$$H(\xi) = (G'/G) \tag{5}$$

where $G = G(\xi)$ satisfies the following auxiliary ordinary differential equation:

$$AGG'' - BGG' - EG^2 - C\,(G')^2 = 0 \tag{6}$$

where the prime stands for derivative with respect to ξ; A, B, C and E are real parameters.

Step 4: To determine the positive integer N, taking the homogeneous balance between the highest order nonlinear terms and the derivatives of the highest order appearing in Eq. (3).

Step 5: Substitute Eq. (4) and Eq. (6) including Eq. (5) into Eq. (3) with the value of N obtained in Step 4, we obtain polynomials in $(d + H)^N \ (N = 0, 1, 2, \cdots)$ and $(d + H)^{-N} \ (N = 0, 1, 2, \cdots)$. Then, we collect each coefficient of the resulted polynomials to zero, yields a set of algebraic equations for $a_i \ (i = 0, 1, 2, \cdots, N)$ and $b_i \ (i = 1, 2, \cdots, N)$, d and V.

Step 6: Suppose that the value of the constants $a_i \ (i = 0, 1, 2, \cdots, N)$, $b_i \ (i = 1, 2, \cdots, N)$, d and V can be found by solving the algebraic equations obtained in Step 5. Since the general solution of Eq. (6) is well known to us, inserting the values of $a_i \ (i = 0, 1, 2, \cdots, N)$, $b_i \ (i = 1, 2, \cdots, N)$, d and V into Eq. (4), we obtain more general type and new exact traveling wave solutions of the nonlinear partial differential equation (1).

Using the general solution of Eq. (6), we have the following solutions of Eq. (5):

When $B \neq 0$, $\psi = A - C$ and $\Omega = B^2 + 4E(A - C) > 0$,

$$H(\xi) = \left(\frac{G'}{G}\right) = \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Omega}}{2A}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\Omega}}{2A}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\Omega}}{2A}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\Omega}}{2A}\xi\right)} \tag{7}$$

When $B \neq 0$, $\psi = A - C$ and $\Omega = B^2 + 4E(A - C) < 0$,

$$H(\xi) = \left(\frac{G'}{G}\right) = \frac{B}{2\psi} + \frac{\sqrt{-\Omega}}{2\psi} \frac{-C_1 \sin\left(\frac{\sqrt{-\Omega}}{2A}\xi\right) + C_2 \cos\left(\frac{\sqrt{-\Omega}}{2A}\xi\right)}{C_1 \cos\left(\frac{\sqrt{-\Omega}}{2A}\xi\right) + C_2 \sin\left(\frac{\sqrt{-\Omega}}{2A}\xi\right)} \tag{8}$$

When $B \neq 0$, $\psi = A - C$ and $\Omega = B^2 + 4E(A - C) = 0$,

$$H(\xi) = \left(\frac{G'}{G}\right) = \frac{B}{2\psi} + \frac{C_2}{C_1 + C_2\xi} \tag{9}$$

When $B = 0$, $\psi = A - C$ and $\Delta = \psi E > 0$,

$$H(\xi) = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Delta}}{\psi} \frac{C_1 \sinh(\frac{\sqrt{\Delta}}{A}\xi) + C_2 \cosh(\frac{\sqrt{\Delta}}{A}\xi)}{C_1 \cosh(\frac{\sqrt{\Delta}}{A}\xi) + C_2 \sinh(\frac{\sqrt{\Delta}}{A}\xi)} \tag{10}$$

When $B=0,\ \psi=A-C$ and $\Delta=\psi E<0,$

$$H(\xi)=\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Delta}}{\psi}\frac{-C_1\sin(\frac{\sqrt{-\Delta}}{A}\xi)+C_2\cos(\frac{\sqrt{-\Delta}}{A}\xi)}{C_1\cos(\frac{\sqrt{-\Delta}}{A}\xi)+C_2\sin(\frac{\sqrt{-\Delta}}{A}\xi)} \tag{11}$$

Application of the method

Let us consider the (3+1)-dimensional Burgers equation,

$$u_t-uu_x-u_{xx}-u_{yy}=0 \tag{12}$$

where, $u=u(x,y,t),\ \xi=k(x+y-Vt),\ u(x,y,t)=u(\xi)$ (13)

By exerting the traveling wave transformation (13) . Equation (12) abates to an ODE

$$-Vu-uu'-2ku''=0 \tag{14}$$

Now integrating the above equation with respect to ξ, we get,

$$2Vu+u^2+4ku'+p=0 \tag{15}$$

where p is an integration constant which is to be ordained.

Balancing the highest order derivative u' and the highest order nonlinear term u^2 in equation (15), we attain $N=1$.Therefore; the solution of equation (15) is of the form

$$u(\xi)=a_0+a_1(d+M)+b_1(d+M)^{-1} \tag{16}$$

where a_0,a_1,b_1 and d are constant to be ordained.

Substituting Eq. (16) together with Eqs. (5) and (6) into Eq. (15), the left-hand side is converted into polynomials in $(d+M)^N$ $(N=0,1,2,.......)$ and $(d+M)^{-N}$ $(N=1,\ 2,\cdots)$. We collect each coefficient of these resulted polynomials to zero yields a set of simultaneous algebraic equations (for simplicity, the equations are not presented) for a_0, a_1, b_1, d, p and V. Solving these algebraic equations with the help of computer algebra, we obtain following:

Set 1:

$$p=\frac{1}{A^2}(16k^2Bd\psi-16k^2E\psi+16k^2C^2d^2-32k^2Cd^2+a_0{}^2A^2$$
$$+16k^2A^2d^2+8a_0AkCd-4a_0AkB-8a_0A^2kd),$$

$$V=\frac{4kd\psi+2kB+4kdA}{A},d=d,k=k,a_0=a_0,a_1=0,\ b_1=-\frac{4k(Bd-E+d^2\psi)}{A}. \tag{17}$$

where $\psi=A-C$, a_0, d,A,B,C,E are free parameters.

Set 2:

$$p=\frac{1}{A^2}(a_0{}^2A^2+16k^2Bd\psi-16k^2E\psi+16k^2C^2d^2-32k^2Cd^2A$$
$$+16k^2A^2d^2+4a_0AkB+8a_0A^2kd-8a_0AkCd),$$

$$V=-\frac{2kB+4kd\psi+a_0A}{A},\ d=d,k=k,b_1=0,\ a_1=\frac{4k\psi}{A}. \tag{18}$$

where, $\psi=A-C$, a_0, d,A,B,C,E are free parameters.

Set 3:

$$p=\frac{a_0{}^2A^2-64k^2E\psi-16k^2B^2}{A^2},\ V=-a_0,\ d=-\frac{B}{2\psi},\ k=k,\ a_0=a_0,$$

$$a_1=\frac{4k\psi}{A},\ b_1=\frac{k(4E\psi+B^2)}{A\psi}. \tag{19}$$

where, $\psi=A-C$, a_0, d,A,B,C,E are free parameters.

For set 1, substituting Eq. (17) into Eq. (16), along with Eq. (7) and simplifying, yields following traveling wave solutions, if $C_1 = 0$ but $C_2 \neq 0$; $C_2 = 0$ but $C_1 \neq 0$ respectively:

$$u_{1,1}(\xi) = a_0 - \frac{4k(Bd - E + d^2\psi)}{A} \times \left(d + \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi} \coth\left(\frac{\sqrt{\Omega}}{2A} \xi \right) \right)^{-1}$$

$$u_{1,2}(\xi) = a_0 - \frac{4k(Bd - E + d^2\psi)}{A} \times \left(d + \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi} \tanh\left(\frac{\sqrt{\Omega}}{2A} \xi \right) \right)^{-1}$$

Similarly, substituting Eq. (17) into Eq. (16), along with Eqs. (8), (9), (10) and (11) and simplifying, the obtained exact solutions become (if $C_1 = 0$ but $C_2 \neq 0$; $C_2 = 0$ but $C_1 \neq 0$) respectively:

$$u_{1,3}(\xi) = a_0 - \frac{4k(Bd - E + d^2\psi)}{A} \times \left(d + \frac{B}{2\psi} + \frac{\sqrt{-\Omega}}{2\psi} \cot\left(\frac{\sqrt{-\Omega}}{2A} \xi \right) \right)^{-1}$$

$$u_{1,4}(\xi) = a_0 - \frac{4k(Bd - E + d^2\psi)}{A} \times \left(d + \frac{B}{2\psi} + \frac{\sqrt{-\Omega}}{2\psi} \tan\left(\frac{\sqrt{-\Omega}}{2A} \xi \right) \right)^{-1}$$

$$u_{1,5}(\xi) = a_0 - \frac{4k(Bd - E + d^2\psi)}{A} \times \left(d + \frac{B}{2\psi} + \frac{C_2}{C_1 + C_2\xi} \right)^{-1}$$

$$u_{1,6}(\xi) = a_0 - \frac{4k(Bd - E + d^2\psi)}{A} \times \left(d + \frac{\sqrt{\Delta}}{\psi} \coth\left(\frac{\sqrt{\Delta}}{A} \xi \right) \right)^{-1}$$

$$u_{1,7}(\xi) = a_0 - \frac{4k(Bd - E + d^2\psi)}{A} \times \left(d + \frac{\sqrt{\Delta}}{\psi} \tanh\left(\frac{\sqrt{\Delta}}{A} \xi \right) \right)^{-1}$$

$$u_{1,8}(\xi) = a_0 - \frac{4k(Bd - E + d^2\psi)}{A} \times \left(d + \frac{\sqrt{-\Delta}}{\psi} \cot\left(\frac{\sqrt{-\Delta}}{A} \xi \right) \right)^{-1}$$

$$u_{1,9}(\xi) = a_0 - \frac{4k(Bd - E + d^2\psi)}{A} \times \left(d + \frac{\sqrt{-\Delta}}{\psi} \tan\left(\frac{\sqrt{-\Delta}}{A} \xi \right) \right)^{-1}$$

where $\xi = x + y - \frac{4kd\psi + 2kB + 4kdA}{A} t$

Again for set 2, substituting Eq. (18) into Eq. (16), along with Eq. (7) and simplifying, our traveling wave solutions become, if $C_1 = 0$ but $C_2 \neq 0$; $C_2 = 0$ but $C_1 \neq 0$ respectively:

$$u_{2,1}(\xi) = a_0 + \frac{4k\psi}{A} \times \left(d + \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi} \coth\left(\frac{\sqrt{\Omega}}{2A} \xi \right) \right)$$

$$u_{2,2}(\xi) = a_0 + \frac{4k\psi}{A} \times \left(d + \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi} \tanh\left(\frac{\sqrt{\Omega}}{2A} \xi \right) \right)$$

Similarly, substituting Eq. (18) into Eq. (16), along with Eqs. (8), (9), (10) and (11) and simplifying, the obtained exact solutions become (if $C_1 = 0$ but $C_2 \neq 0$; $C_2 = 0$ but $C_1 \neq 0$) respectively:

$$u_{2,3}(\xi) = a_0 + \frac{4k\psi}{A} \times \left(d + \frac{B}{2\psi} + i \frac{\sqrt{\Omega}}{2\psi} \cot\left(\frac{\sqrt{-\Omega}}{2A} \xi \right) \right)$$

$$u_{2,4}(\xi) = a_0 + \frac{4k\psi}{A} \times \left(d + \frac{B}{2\psi} - i \frac{\sqrt{\Omega}}{2\psi} \tan\left(\frac{\sqrt{-\Omega}}{2A} \xi \right) \right)$$

$$u_{2,5}(\xi)=a_0+\frac{4k\psi}{A}\times\left(d+\frac{B}{2\psi}+\frac{C_2}{C_1+C_2\xi}\right)$$

$$u_{2,6}(\xi)=a_0+\frac{4k\psi}{A}\times\left(d+\frac{\sqrt{\Delta}}{\psi}\coth\left(\frac{\sqrt{\Delta}}{A}\xi\right)\right)$$

$$u_{2,7}(\xi)=a_0+\frac{4k\psi}{A}\times\left(d+\frac{\sqrt{\Delta}}{\psi}\tanh\left(\frac{\sqrt{\Delta}}{A}\xi\right)\right)$$

$$u_{2,8}(\xi)=a_0+\frac{4k\psi}{A}\times\left(d+\frac{i\sqrt{\Delta}}{\psi}\cot\left(\frac{\sqrt{-\Delta}}{A}\xi\right)\right)$$

$$u_{2,9}(\xi)=a_0+\frac{4k\psi}{A}\times\left(d-\frac{i\sqrt{\Delta}}{\psi}\tan\left(\frac{\sqrt{-\Delta}}{A}\xi\right)\right)$$

where $\xi=x+y+\frac{2kB+4kd\psi+a_0A}{A}t$

Again for set 3, substituting Eq. (19) into Eq. (16), together with Eq. (7) and simplifying, yields following traveling wave solutions, if $C_1=0$ but $C_2\neq 0$; $C_2=0$ but $C_1\neq 0$ respectively:

$$u_{3,1}(\xi)=a_0+\frac{2k}{A}\left(\sqrt{\Omega}\times\coth\left(\frac{\sqrt{\Omega}}{2A}\xi\right)+\frac{1}{\sqrt{\Omega}}\left(4E\psi+B^2\right)\times\tanh\left(\frac{\sqrt{\Omega}}{2\psi}\xi\right)\right)$$

$$u_{3,1}(\xi)=a_0+\frac{2k}{A}\left(\sqrt{\Omega}\times\tanh\left(\frac{\sqrt{\Omega}}{2A}\xi\right)+\frac{1}{\sqrt{\Omega}}\left(4E\psi+B^2\right)\times\coth\left(\frac{\sqrt{\Omega}}{2\psi}\xi\right)\right)$$

Similarly, substituting Eq. (18) into Eq. (16), along with Eqs. (8), (9), (10) and (11) and simplifying, the obtained exact solutions become (if $C_1=0$ but $C_2\neq 0$; $C_2=0$ but $C_1\neq 0$) respectively:

$$u_{3,4}(\xi)=a_0+\frac{2k}{A}\left(i\sqrt{\Omega}\times\cot\left(\frac{\sqrt{-\Omega}}{2A}\xi\right)+\frac{1}{i\sqrt{\Omega}}\left(4E\psi+B^2\right)\times\tan\left(\frac{\sqrt{-\Omega}}{2\psi}\xi\right)\right),$$

$$u_{3,4}(\xi)=a_0-\frac{2k}{A}\left(i\sqrt{\Omega}\times\tan\left(\frac{\sqrt{-\Omega}}{2A}\xi\right)+\frac{1}{i\sqrt{\Omega}}\left(4E\psi+B^2\right)\times\cot\left(\frac{\sqrt{-\Omega}}{2\psi}\xi\right)\right),$$

$$u_{3,5}(\xi)=a_0+\frac{4k\psi}{A}\times\left(\frac{C_2}{C_1+C_2\xi}\right)+\frac{K\left(4E\psi+B^2\right)}{A\psi}\times\left(\frac{C_2}{C_1+C_2\xi}\right)^{-1},$$

$$u_{3,6}(\xi)=a_0+\frac{4k\psi}{A}\times\left(-\frac{B}{2\psi}+\frac{\sqrt{\Delta}}{\psi}\coth\left(\frac{\sqrt{\Delta}}{A}\xi\right)\right)+\frac{K\left(4E\psi+B^2\right)}{A\psi}\times\left(-\frac{B}{2\psi}+\frac{\sqrt{\Delta}}{\psi}\coth\left(\frac{\sqrt{\Delta}}{A}\xi\right)\right)^{-1}$$

$$u_{3,7}(\xi)=a_0+\frac{4k\psi}{A}\times\left(-\frac{B}{2\psi}+\frac{\sqrt{\Delta}}{\psi}\tanh\left(\frac{\sqrt{\Delta}}{A}\xi\right)\right)+\frac{K\left(4E\psi+B^2\right)}{A\psi}\times\left(-\frac{B}{2\psi}+\frac{\sqrt{\Delta}}{\psi}\tanh\left(\frac{\sqrt{\Delta}}{A}\xi\right)\right)^{-1}$$

$$u_{3,8}(\xi)=a_0+\frac{4k\psi}{A}\times\left(-\frac{B}{2\psi}+\frac{\sqrt{-\Delta}}{\psi}\cot\left(\frac{\sqrt{-\Delta}}{A}\xi\right)\right)$$
$$+\frac{k\left(4E\psi+B^2\right)}{A\psi}\times\left(-\frac{B}{2\psi}+\frac{\sqrt{-\Delta}}{\psi}\cot\left(\frac{\sqrt{-\Delta}}{A}\xi\right)\right)^{-1}$$

$$u_{3,9}(\xi)=a_0+\frac{4k\psi}{A}\times\left(-\frac{B}{2\psi}-\frac{\sqrt{-\Delta}}{\psi}\tan\left(\frac{\sqrt{-\Delta}}{A}\xi\right)\right)$$
$$+\frac{k\left(4E\psi+B^2\right)}{A\psi}\times\left(-\frac{B}{2\psi}-\frac{\sqrt{-\Delta}}{\psi}\tan\left(\frac{\sqrt{-\Delta}}{A}\xi\right)\right)^{-1}$$

where $\xi=x+y+a_0t$.

Remark 1: Some of these solutions presented in this latter have been checked with Maple by putting them back into the original equations.

Remark 2: The new generalized (G'/G)-expansion method is simple but its results are very cumbersome. The results of this method contain many arbitrary constants compare to the results of the other method. The performance of new generalized (G'/G)-expansion method is reliable, simple, direct, concise and gives more new exact solutions compared to the other method. This method allowed us to solve more complicated PDEs in the mathematical physics.

NUMERICAL RESULTS AND DISCUSSION

The introduction of dispersion without introducing nonlinearity destroys the solitary wave as different Fourier harmonics start propagating at different group velocities. On the other hand, introducing nonlinearity without dispersion also prevents the formation of solitary waves, because the pulse energy is frequently pumped into higher frequency modes. However, if both dispersion and nonlinearity are present, solitary waves can be sustained. Similarly to dispersion, dissipation can also give rise to solitary waves when combined with nonlinearity. Hence it is interesting to point out that the delicate balance between the nonlinearity effect of uu_x and the dissipative effect of u_{xx} and u_{yy} give rise to solitons, that after a fully interaction with others the solitons come back retaining their identities with the same speed and shape. The (2+1)-dimensional Burgers equation has solitary wave solutions that have exponentially decaying wings. If two solitons of the (2+1)-dimensional Burgers equation collide, the solitons just pass through each other and emerge unchanged. For special values of the parameters solitary wave solutions are originated from the obtained exact solutions.

Fig. 1: Kink solution of $u_{1_2}(\eta)$ when $d=1,\ a_0=1,\ A=4,\ B=1,\ k=1,\ C=1,\ E=1$ within the interval $-10 \le x,t \le 10$. Solutions $u_{1_7}(\eta)$, $u_{2_2}(\eta)$ and $u_{2_7}(\eta)$ represent kink. Kink waves are traveling waves which arise from one asymptotic state to another. The kink solutions are approach to a constant at infinity. Other figures are omitted for convenience.

Fig. 2: Singular Kink solution of $u_{2_5}(\eta)$ when $a_0=1,\ d=1,\ k=1,\ A=1,\ B=2,\ C=2,\ C_1=2,\ C_2=1,\ E=1$ within the interval $-10 \le x,t \le 10$. Solutions $u_{1_1}(\eta)$, $u_{1_5}(\eta)$, $u_{1_6}(\eta)$ $u_{3_5}(\eta)$ represent singular Kink solution. Other figures are omitted for convenience.

Fig. 3: Periodic solutions of $u_{1_4}(\eta)$ when $a_0=1,\ d=5,\ A=2,\ B=1,\ k=1,\ C=3,\ E=1$ within the interval $-1 \le x,t \le 1$. Solutions $u_{1_3}(\eta)$, $u_{1_8}(\eta)$, $u_{1_9}(\eta)$ $u_{2_4}(\eta)$ $u_{2_8}(\eta)$ $u_{29}(\eta)$ represent the exact periodic traveling wave solutions. Periodic solutions are traveling wave solutions that are periodic such as $\cos(x-t)$. Other figures are omitted for convenience.

Fig. 4: Singular soliton solutions of $u_{2_3}(\eta)$ when $a_0=5,\ A=2,\ B=1,\ d=1,\ k=1,\ C=4,\ E=1$ within the interval $-1 \le x,t \le 1$. Solutions $u_{2_1}(\eta)$, $u_{2_6}(\eta)$, $u_{3_3}(\eta)-u_{3_4}(\eta)$, $u_{3_6}(\eta)-u_{3_9}(\eta)$ are called the singular soliton solution. Other figures are omitted for convenience.

Fig. 5: Soliton solution of $u_{3_1}(\eta)$ when $k=1,\ a_0=1,\ A=4,\ B=1,\ C=1,\ E=1$ and $-10 \le x,t \le 10$. Solutions $u_{3_1}(\eta)$ and $u_{3_2}(\eta)$ describe the soliton. Solitons are special kinds of solitary waves. The soliton solution is a specially localized solution, hence $u'(\xi),\ u''(\xi),\ u'''(\xi) \to 0$ as $\xi \to \pm\infty$, $\xi = x - ct$. Solitons have a remarkable property that it keeps its identity upon interacting with other solitons.

Fig. 6: Singular soliton solution of $u_{3_9}(\eta)$ when $a_0=1,\ k=1,\ A=1,\ B=0,\ C=2,\ E=2$ and $-10 \le x,t \le 10$.

Graphical representation

Some of our obtained traveling wave solutions are represented in the following figures with the aid of commercial software Maple 13:

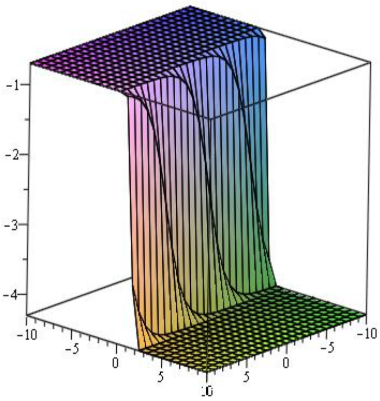

Fig. 1. Kink solution of $u_{1_2}(\eta)$ when $d=1$, $a_0=1$, $A=4$, $B=1$, $k=1$, $C=1$, $E=1$ and $-10\le x,t\le 10$.

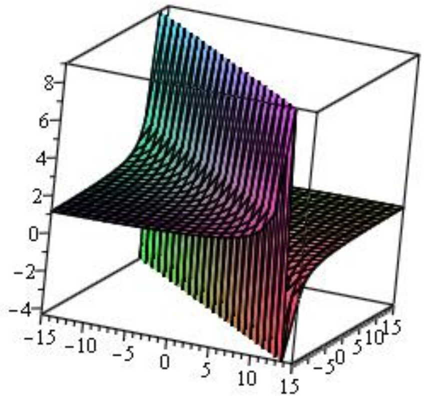

Fig. 2. Singular Kink solution of $u_{2_5}(\eta)$ when $a_0=1$, $d=1$, $k=1$, $A=1$, $B=2$, $C=2$, $C_1=2$, $C_2=1$, $E=1$ and $-10\le x,t\le 10$.

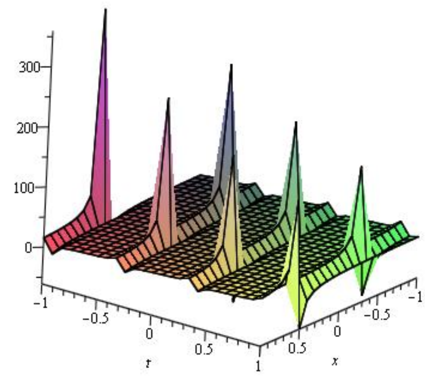

Fig. 3. Periodic solutions of $u_{1_4}(\eta)$ when $a_0=1$, $d=5$, $A=2$, $B=1$, $k=1$, $C=3$, $E=1$ and $-1\le x,t\le 1$.

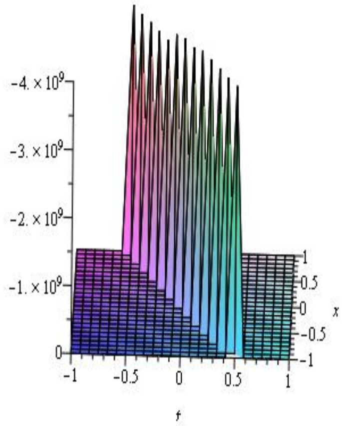

Fig. 4. Singular soliton solutions of $u_{2_3}(\eta)$ when $a_0=5$, $A=2$, $B=1$, $d=1$, $k=1$, $C=4$, $E=1$ and $-1\le x,t\le 1$.

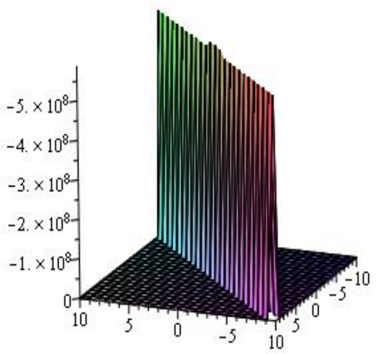

Fig. 5. Soliton solution of $u_{3_1}(\eta)$ when $k=1$, $a_0=1$, $A=4$, $B=1$, $C=1$, $E=1$ and $-10\le x,t\le 10$.

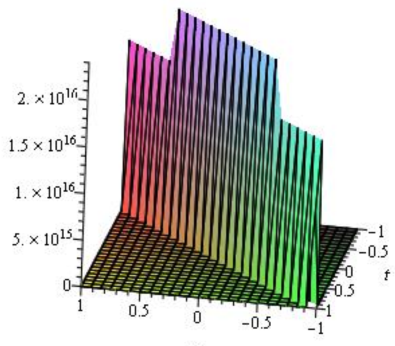

Fig. 6. Singular soliton solution of $u_{3_9}(\eta)$ when $a_0=1$, $k=1$, $A=1$, $B=0$, $C=2$, $E=2$ and $-10\le x,t\le 10$.

CONCLUSION

The new generalized (G'/G)-expansion method presented in this paper has been successfully implemented to construct many new and more general exact solutions of the (2+1)-dimensional Burgers equation. The method offers solutions with free parameters that might be important to explain some complex physical phenomena. This study shows that the new generalized (G'/G)-expansion method is quite efficient and practically well suited to be used in finding exact solutions of NLEEs.

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