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WATER WAVES AND SIGNIFICANCE OF WAVE HEIGHT IN THE BAY OF BENGAL

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ABSTRACT

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This paper presents a method to estimate the water waves and significant wave height from the observed data. For the estimation of extreme water waves and significant wave height, a precise representation of the data by a certain probability function is highly desirable. Since we do not have any specific technique to meet this requirement, this situation seriously affects the reliability of the current method. It expresses asymptotically the cumulative distribution of the water waves and significant wave height as a combination of an exponential and power of the water waves and significant wave height. The parameters involved are determined numerically by a nonlinear minimization procedure. The results of the analysis show that the data are well represented by the proposed method over the entire range of the cumulative distribution.

Key words: *water waves, significant wave height, nonlinear, wave energy*

INTRODUCTION

A wave can be described as a disturbance that travels through a medium from one location to another location. Wave's disturbances of water are a constant presence in the world's oceans. Since waves travel all across the globe, transmitting vast amounts of energy, understanding their motions and characteristics is essential. The forces generated by waves are the main factor impacting the geometry of beaches, the transport of sand and other sediments in the near shore region, and the stresses and strains on coastal structures. When waves are large, they can also pose a significant threat to commercial shipping, recreational boaters, and the beach going public. In the open river, ocean, etc. waves are formed when winds blowing across the water surface transfer energy to the water. This energy is then transmitted from the region of wave generation as a water wave. In a water wave, the water particles travel in circular orbits. With depth the size of the orbits decreases, and at a depth equal to about one-half the wavelength, the orbital diameters are only about 1/25 of those at the surface (Anonymous, Undated[a]). For all practical purposes, we may consider this level as the maximum depth of wave motion. The distance between successive crests (or troughs) is the wavelength. The time it takes two crests (or troughs) to pass a point is the wave period. The displacement of the water surface from the rest position (flat surface) is the amplitude and the distance from wave trough to wave crest is the wave height. When the waves are under the direct influence of the wind, they tend to have round troughs and peaked crests. Such waves are referred to as sea. When the waves move out of the area of wave generation, and are no longer under the direct influence of the wind, they take on a more rounded (sinusoidal) shape (Anonymous, Undated[b]). Such waves are referred to as swell. When waves enter shallow water they may begin to interact with the bottom. When this occurs the shape of the water particle orbital's change, the waves slow down and steepen. It is said that the waves "feel the bottom" and the depth at which this occurs is called the wave base. Under these conditions the waves can move bottom sediment.

The significant wave height (SWH or H_s) is defined traditionally as the mean wave height (trough to crest) of the highest third of the waves ($H_{1/3}$) (Anonymous, Undated[c]). Nowadays it is usually defined as four times the standard deviation of the surface elevation – or equivalently as four times the square root of the zeroth-order moment (area) of the wave spectrum (Anonymous, Undated[d]). The symbol H_{m0} is usually used for that latter definition. The significant wave height may thus refer to H_{m0} or $H_{1/3}$; the difference in magnitude between the two definitions is only a few percent. Significant wave height, scientifically represented as H_s or H_{sig}, is an important parameter for the statistical distribution of ocean waves. The most common waves are less in height than H_s. This implies that encountering the significant wave is not too frequent. However, statistically, it is possible to encounter a wave that is much higher than the significant wave. Although most measuring devices estimate the significant wave height from a wave spectrum, satellite radar altimeters are unique in measuring directly the significant wave height thanks to the different time of return from wave crests and troughs within the area illuminated by the radar.

MATERIALS AND METHODS

Deep water waves

As a rule, when the water depth is greater than one-half of the wavelength, the wave is classified as a deepwater wave (Anonymous, Undated[e]). Water particles inside this type of wave move forward and back, up and down in a circular orbit whose diameter decreases with depth until it essentially disappear at the wave base. With each orbit, the particles inch their way forward by a slight amount producing what is called Stokes drift (Anonymous, Undated[e]). The waveform itself moves forward in the direction of wave advance at a much faster rate.

Shallow water waves

When waves travel into areas of shallow water, they begin to be affected by the ocean bottom. The free orbital motion of the water is disrupted, and water particles in orbital motion no longer return to their original position. As the water becomes shallower, the swell becomes higher and steeper, ultimately assuming the familiar sharp-crested wave shape. After the wave breaks, it becomes a wave of translation and erosion of the ocean bottom intensifies.

Shallow water wave equation or **Shallow water gravity wave equation**

The adjustment under gravity, a homogeneous shallow layer of fluid will be considered for a rotating fluid motion. The bottom z = - H and the surface elevation z = η is assumed to be small.

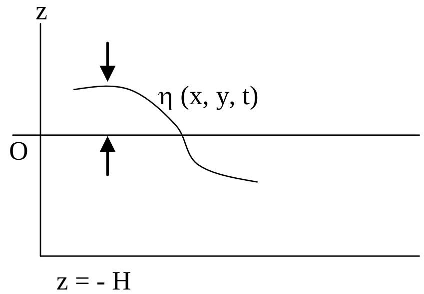

Fig. 1. Perturbation from the rest state for a homogeneous fluid

The free surface is at z = 0 and the bottom is at z = - H. The equilibrium pressure P_o (z) in this case is given by

$$P_o(z) = -\rho g z$$

It is convenient to define the perturbation pressure by

$$P = p_o + p'$$

The pressure must be vanish at free surface

$$\text{i.e} \quad P = -\rho g z + p' = 0$$
$$\Rightarrow p' = \rho g z$$
$$\text{i.e} \quad p' = \rho g z \text{ at } z = \eta \; \dots \dots \dots \text{(i)}$$

The momentum equation can be written as–

$$\frac{\delta u}{\delta t} - fv = -\frac{1}{\rho}\frac{\delta p'}{\delta x} = -g\frac{\delta \eta}{\delta x} \; \dots \dots \dots \text{(ii)} \quad \text{[by (i)]}$$

$$\frac{\delta v}{\delta t} + fu = -\frac{1}{\rho}\frac{\delta p'}{\delta y} = -g\frac{\delta \eta}{\delta y} \; \dots \dots \dots \text{(iii)}$$

The equation of continuity for in viscid incompressible fluid $\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0$ can be integrated with respect to depth using the boundary conditions.

$$w = 0 \text{ at } z = -H$$

$$\text{and} \quad w = \frac{\delta \eta}{\delta t} \text{ at } z = \eta$$

$$\text{i.e} \int \left(\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z}\right) dz = \text{constant}$$

$$\Rightarrow \left(\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y}\right) z + w = \text{constant}$$

using the boundary conditions. We have

$$\left(\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y}\right)(-H) = \text{constant} \; \dots \dots \dots \text{(A)}$$

$$\text{and} \left(\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y}\right)\eta + \frac{\delta \eta}{\delta t} = \text{constant} \; \dots \dots \dots \text{(B)}$$

Subtracting (A) from (B) we get

$$\left(\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}\right)(H+\eta)+\frac{\delta \eta}{\delta t}=0$$

$$\Rightarrow \frac{\delta \eta}{\delta t}+\left(\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}\right)(H+\eta)=0$$

Since η is very small, then

$$\frac{\delta \eta}{\delta t}+\left(\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}\right)H=0 \quad \cdots\cdots\cdots \text{(iv)}$$

Differentiating (ii) partially w. r. to x and (iii) w. r. to y, then adding together, we get,

$$\frac{\delta}{\delta t}\left(\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}\right)-f\left(\frac{\delta v}{\delta x}-\frac{\delta u}{\delta y}\right)=-g\left(\frac{\delta^2 \eta}{\delta x^2}+\frac{\delta^2 \eta}{\delta y^2}\right)$$

$$\Rightarrow \frac{\delta}{\delta t}\left(-\frac{\delta \eta}{\delta t}/H\right)-f\zeta=-g\left(\frac{\delta^2 \eta}{\delta x^2}+\frac{\delta^2 \eta}{\delta y^2}\right)$$

[using (iv) & $\zeta=\left(\frac{\delta v}{\delta x}-\frac{\delta u}{\delta y}\right)$ from vorticity]

$$\Rightarrow \frac{\delta^2 \eta}{\delta t^2}+fH\zeta=gH\left(\frac{\delta^2 \eta}{\delta x^2}+\frac{\delta^2 \eta}{\delta y^2}\right)$$

$$\Rightarrow \frac{\delta^2 \eta}{\delta t^2}-gH\left(\frac{\delta^2 \eta}{\delta x^2}+\frac{\delta^2 \eta}{\delta y^2}\right)+fH\zeta=0$$

$$\Rightarrow \frac{\delta^2 \eta}{\delta t^2}-c^2\left(\frac{\delta^2 \eta}{\delta x^2}+\frac{\delta^2 \eta}{\delta y^2}\right)+fH\zeta=0 \quad \cdots\cdots\cdots (*)$$

Where $c^2 = gH$ and $\zeta = \frac{\delta v}{\delta x}-\frac{\delta u}{\delta y}$ is the relative velocity of the fluid i.e vertical component of the vortices relative to the rotating frame.

When f = 0, then (*) reduces to the wave equation.

$$\frac{\delta^2 \eta}{\delta t^2}=c^2\left(\frac{\delta^2 \eta}{\delta x^2}+\frac{\delta^2 \eta}{\delta y^2}\right)=c^2\nabla^2\eta$$

Which is the required shallow water wave equation.

Sinusoidal waves

In linear media, any wave pattern can be described in terms of the independent propagation of sinusoidal components. The wavelength λ of a sinusoidal waveform traveling at constant speed v is given by:

$$v=\lambda f\ ,$$

where v is called the phase speed (magnitude of the phase velocity) of the wave and f is the wave's frequency. In a dispersive medium, the frequency depends upon the wavelength of the wave, and accordingly waves with different λ in general will travel with a different speed v.

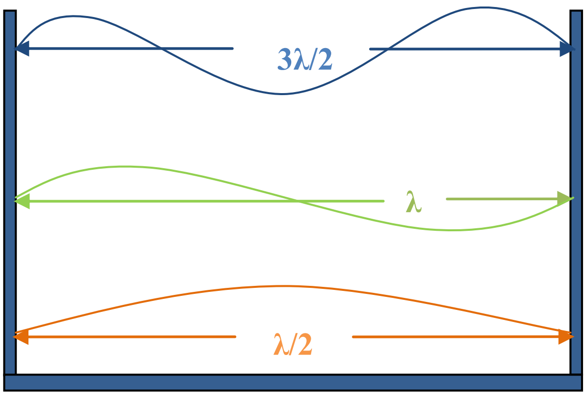

Fig. 2. Sinusoidal wave

- Sinusoidal standing waves in a box that constrains the end points to be nodes will have an integer number of half wavelengths fitting in the box.

Standing waves

A standing wave is an undulatory motion that stays in one place. A sinusoidal standing wave includes stationary points of no motion, called nodes, and the wavelength is twice the distance between nodes. Figure 2 shows three standing waves in a box. The walls of the box are considered to require the wave have nodes at the walls of the box (an example of boundary conditions) determining which wavelengths are allowed. For example, for an electromagnetic wave, if the box has ideal metal walls, the condition for nodes at the walls results because the metal walls cannot support a tangential electric field, forcing the wave to have zero amplitude at the wall.

The stationary wave can be viewed as the sum of two traveling sinusoidal waves of oppositely directed velocities. Consequently, wavelength, period, and wave velocity are related just as for a traveling wave. For example, the speed of light can be determined from observation of standing waves in a metal box containing an ideal vacuum.

Mathematical representation

Traveling sinusoidal waves are often represented mathematically in terms of their velocity v (in the x direction), frequency f and wavelength λ as:

$$y(x,t) = A\cos\left(2\pi\left(\frac{x}{\lambda} - ft\right)\right) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

Where y is the value of the wave at any position x and time t, and A is the amplitude of the wave. They are also commonly expressed in terms of (radian) wave number k (2π times the reciprocal of wavelength) and angular frequency ω (2π times the frequency) as:

$$y(x,t) = A\cos(kx - \omega t) = A\cos\big(k(x - vt)\big)$$

in which wavelength and wave number are related to velocity and frequency as:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v}$$

Or

$$\lambda = \frac{2\pi}{k} = \frac{2\pi v}{\omega} = \frac{v}{f}$$

Wave propagation and dispersion

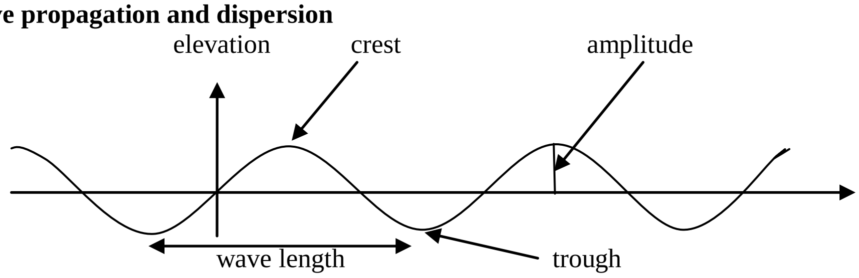

Fig. 3. Ocean characteristics (Sathiya and Vaithiyanathan, 2012)

The simplest propagating wave of unchanging form is a sine wave. A sine wave with water surface elevation $\eta(x, t)$ is given by:

$$\eta(x,t) = a\sin\big(\theta(x,t)\big),$$

where a is the amplitude (in metres) and $\theta = \theta(x, t)$ is the phase function (in radians), depending on the horizontal position (x, in metres) and time (t, in seconds):

$$\theta = 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) = kx - \omega t, \text{ with } k = \frac{2\pi}{\lambda} \text{ and } \omega = \frac{2\pi}{T}$$

Where:

- λ is the wavelength (in metres),
- T is the period (in seconds),
- k is the wave number (in radians per metre) and
- ω is the angular frequency (in radians per second).

Characteristic phases of a water wave are:

- the upward zero-crossing at $\theta = 0$,
- the wave crest at $\theta = ½\,\pi$,
- the downward zero-crossing at $\theta = \pi$ and
- the wave trough at $\theta = 1½\,\pi$.

A certain phase repeats itself after an integer *m* multiple of 2π: sin (θ) = sin ($\theta + m \bullet 2\pi$).

Essential for water waves, and other wave phenomena in physics, is that free propagating waves of non-zero amplitude only exist when the angular frequency ω and wave number *k* (or equivalently the wavelength λ and period *T*) satisfy a functional relationship: the frequency dispersion relation

$$\omega^2 = \Omega^2(k)$$

The dispersion relation has two solutions: $\omega = +\Omega\ (k)$ and $\omega = -\Omega\ (k)$, corresponding to waves travelling in the positive or negative *x*–direction. The dispersion relation will in general depend on several other parameters in addition to the wave number *k*. For gravity waves, according to linear theory, these are the acceleration by gravity and the water depth.

An initial wave phase $\theta = \theta_0$ propagates as a function of space and time. Its subsequent position is given by:

$$x = \frac{\lambda}{T}t + \frac{\lambda}{2\pi}\theta_o = \frac{\omega}{k}t + \frac{\theta_o}{k}$$

This shows that the phase moves with the velocity:

$$c_p = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{\Omega(k)}{k},$$

which is called the phase velocity.

Linear Theory of Ocean Surface Waves

Surface waves are inherently nonlinear: The solution of the equations of motion depends on the surface boundary conditions, but the surface boundary conditions are the waves.

We assume that the amplitude of waves on the water surface is infinitely small so that the surface is almost exactly a plane. To simplify the mathematics, we can also assume that the flow is two-dimensional with waves traveling in the x-direction. We also assume that the Coriolis force and viscosity can be neglected.

With these assumptions, the sea-surface elevation ζ of a wave traveling in the *x* direction is:

$$\xi = a \sin(kx - \omega t)$$

With

$$\omega = 2\pi f = 2\pi T\ ;\ \ k = 2\pi L$$

where ω is wave frequency in radians per second, *f* is the wave frequency in hertz (Hz), *k* is wave number, *T* is wave period, *L* is wave length, and where we assume, as stated above, that $ka = O(0)$.

The wave period *T* is the time it takes two successive wave crests or troughs to pass a fixed point. The wavelength *L* is the distance between two successive wave crests or troughs at a fixed time.

Dispersion Relation

Wave frequency ω is related to wave number *k* by the dispersion relation

$$\omega^2 = gk \tan h(kd)$$

where *d* is the water depth and *g* is the acceleration of gravity.

Two approximations are especially useful.

1. *Deep-water approximation* is valid if the water depth *d* is much greater than the wave length *L*. In this case, $d \gg L$, $kd \gg 1$ and $\tan h(kd)\ = 1$.
2. *Shallow-water approximation* is valid if the water depth is much less than a wave length. In this case, $d \ll L$, $kd \ll 1$ and $\tan h(kd)\ = kd$.

For these two limits of water depth compared with wave length the dispersion Relation reduces to:

$\omega^2 = gk$ Deep-water dispersion relation … … … … (v)

$d > L/4$

$\omega^2 = gk^2d$ Shallow-water dispersion relation … … … (vi)

$d < L/11$

The stated limits for *d/L* give a dispersion relation accurate within 10%. Because many wave properties can be measured with accuracies of 5–10%, the approximations are useful for calculating wave properties.

Phase Velocity

The phase velocity c is the speed at which a particular phase of the wave propagates, for example, the speed of propagation of the wave crest.

In one wave period T the crest advances one wave length L and the phase speed is $c = L/T = \omega/k$. Thus, the definition of phase speed is:

$$c \equiv \frac{\omega}{k}$$

The direction of propagation is perpendicular to the wave crest and toward the positive x -direction.
The deep-and shallow-water approximations for the dispersion relation give:

$$c = \sqrt{\frac{g}{k}} = \frac{g}{w} \quad \text{Deep-water phase velocity}$$

$$c = \sqrt{gd} \quad \text{Shallow-water phase velocity}$$

The approximations are accurate to about 5% for limits stated in (v, vi). In deep water, the phase speed depends on wave length or wave frequency.

Longer waves travel faster. Thus, deep-water waves are said to be dispersive.
In shallow water, the phase speed is in dependent of the wave; it depends only on the depth of the water. Shallow-water waves are non-dispersive.

Group Velocity

The concept of group velocity c_g is fundamental for under-standing the propagation of linear and nonlinear waves. First, it is the velocity at which a group of waves travels across the ocean. More importantly, it is also the propagation velocity of wave energy.

The definition of group velocity in two dimensions is:

$$c_g \equiv \frac{\partial w}{\partial k}$$

Using the approximations for the dispersion relation:

$$c_g = \frac{g}{2w} = \frac{c}{2} \quad \text{Deep-water group velocity}$$

$$c_g = \sqrt{gd} = c \quad \text{Shallow-water group velocity}$$

For ocean-surface waves, the direction of propagation is perpendicular to the wave crests in the positive x direction. In the more general case of other types of waves, such as Kelvin and Rossby waves, the group velocity is not necessarily in the direction perpendicular to wave crests.

Notice that a group of deep-water waves moves at half the phase speed of the waves making up the group. How can this happen? If we could watch closely a group of waves crossing the sea, we would see waves crests appear at the back of the wave train, move through the train, and disappear at the leading edge of the group. Each wave crest moves at twice the speed of the group.

Real ocean waves move in groups governed by the dispersion relation. Munk *et al*. (1963) in a remarkable series of experiments in the 1960s showed that ocean waves propagating over great distances are dispersive and that the dispersion could be used to track storms.

Wave Energy

Wave energy E in joules per square meter is related to the variance of sea-surface displacement ξ by:

$$E = \rho_w g \langle \xi^2 \rangle$$

where ρ_w is water density, g is gravity, and the brackets denote a time or space average.

Significant Wave Height

If we look at a wind-driven sea, we see waves of various heights. Some are much larger than most, others are much smaller. A practical definition that is often used is the height of the highest 1/3 of the waves, $H_{1/3}$. The height is computed as follows: measure wave height for a few minutes, pick out say 120 wave crests and record their heights. Pick the 40 largest waves and calculate the average height of the 40 values. This is $H_{1/3}$ for the wave record.

The concept of significant wave height was developed during the World War II as part of a project to forecast ocean wave heights and periods.

Significant wave height is calculated from measured wave displacement. If the sea contains a narrow range of wave frequencies, $H_{1/3}$ is related to the standard deviation of sea-surface displacement (NAS 1963; Hoffman and Karst, 1975).

$$H_{1/3} = 4\langle\xi^2\rangle^{1/2}$$

where $\langle\xi^2\rangle^{1/2}$ is the standard deviation of surface displacement. This relationship is much more useful, and it is now the accepted way to calculate wave height from wave measurements.

Nonlinear waves

We derived the properties of an ocean surface wave assuming waves were infinitely small $ka = O(0)$. If the waves are small $ka \ll 1$ but not infinitely small, the wave properties can be expanded in a power series of ka (Stokes, 1847). He calculated the properties of a wave of finite amplitude and found:

$$\xi = a\cos(kx - \omega t) + \frac{1}{2}ka^2\cos 2(kx - \omega t) + \frac{3}{8}k^2a^3\cos 3(kx - \omega t) + \ldots. \quad \ldots \quad \ldots \quad \text{(vii)}$$

The phases of the components for the Fourier series expansion of ξ in (vii) are such that non-linear waves have sharpened crests and flattened troughs. The maximum amplitude of the Stokes wave is a_{max} =0.07L (ka =0.44). Such steep waves in deep water are called Stokes waves.

Knowledge of non-linear waves came slowly until Hasselmann (1961, 1963a, 1963b, 1966), using the tools of high-energy particle physics, worked out to 6th Order the interactions of three or more waves on the sea surface. He, Phillips (1960), and Longuet-Higgins and Phillips (1962) showed that n free waves on The sea surface can interact to produce another free wave only if the frequencies And wave numbers of the interacting waves sum to zero:

$$\omega_1 \pm \omega_2 \pm \omega_3 \pm \cdots \omega_n = 0 \quad \ldots \ldots \ldots \quad \text{(viii.a)}$$

$$k_1 \pm k_2 \pm k_3 \pm \cdots k_n = 0 \quad \ldots \ldots \ldots \quad \text{(viii.b)}$$

$$\omega_i^2 = gk_i \quad \ldots \ldots \ldots \quad \text{(viii.c)}$$

where we allow waves to travel in any direction, and k_i is the vector wave number giving wave length and direction. (viii.a,b) are general requirements for any interacting waves. The fewest number of waves that meet the conditions of (viii.a,b,c) are three waves which interact to produce a fourth. The interaction is weak; waves must interact for hundreds of wave lengths and periods to produce a fourth wave with amplitude comparable to the interacting waves. The Stokes wave does not meet the criteria of (viii.a,b,c) and the wave components are not free waves; the higher harmonics are bound to the primary wave.

Wave Momentum

The concept of wave momentum has caused considerable Confusion (McIntyre 1981). In general, waves do not have momentum, a mass flux, but they do have a momentum flux. This is true for waves on the sea surface. Ursell (1950) showed that oceans well on a rotating Earth have no mass transport. His proof seems to contradict the usual text book discussions of steep, non-linear waves such as Stokes waves. Water particles in a Stokes wave move along paths that are nearly circular, but the paths fail to close, and the particles move slowly in the direction of wave propagation. This is a mass transport, and the phenomena are called Stokes drift. But the forward transport near the surface is balanced by an equal transport in the opposite direction at depth, and there is no net mass flux.

Solitary Waves

Solitary waves are another class of non-linear waves, and they have very interesting properties. They propagate without change of shape, and two solitons can cross without interaction.

The properties of a solitary waves result from an exact balance between dispersion which tends to spread the solitary wave into a train of waves, and non-linear effects which tend to shorten and steepen the wave. The type of solitary wave in shallow water seen by Russel has the form:

$$\xi = a\sec h^2\left[\left(\frac{3a}{4d^3}\right)^{1/2}(x - ct)\right]$$

which propagates at a speed:

$$c = c_o\left(1 + \frac{a}{2d}\right)$$

All shallow-water waves are solitons because they are non-dispersive, and hence they ought to propagate without changing shape. Unfortunately, this is not true if the waves have finite amplitude. The velocity of the wave depends on depth. If the wave consists of a single hump, then the water at the crest travels faster than water in the trough, and the wave steepens as it moves forward. Eventually, the wave becomes very steep and breaks. At this point it is called a bore. In some river mouths, the incoming tide is so high and the estuary so long and shallow that the tidal wave entering the estuary eventually steepens and breaks producing a bore that runs up the river.

RESULTS AND DISCUSSION

Our understanding of ocean waves, their spectra, their generation by the wind, and their interactions are now sufficiently well understood that the wave spectrum can be forecast using winds calculated from numerical weather models. If we observe some small ocean area, or some area just offshore, we can see waves generated by the local wind, the wind sea, plus waves that were generated in other areas at other times and that have propagated into the area we are observing, the swell. Forecasts of local wave conditions must include both sea and swell; hence wave forecasting is not a local problem.

Various techniques have been used to forecast waves. The earliest attempts were based on empirical relationships between wave-height and wave-length and wind speed, duration, and fetch. The development of the wave spectrum allowed evolution of individual wave components with frequency f traveling in direction θ of the directional wave spectrum ψ (f,θ) using

$$\frac{\partial \psi_0}{\partial t} + \mathbf{c_g} \cdot \nabla \psi_0 = S_i + S_{nl} + S_d \qquad \text{(ix)}$$

where $\psi_0 = \psi_0$ (f,θ; x, t) varies in space (x) and time t, S_i is input from the wind given by the Phillips (1957) and Miles (1957) mechanisms, S_{nl} is the transfer among wave components due to nonlinear interactions and S_d is dissipation.

The third-generation wave-forecasting models now used by meteorological agencies throughout the world are based on integrations of (16.39) using many individual wave components (The SWAMP Group 1985; The WAMDI Group, 1988; Komen *et al*. 1994). The forecasts follow individual components of the wave spectrum in space and time, allowing each component to grow or decay depending on local winds, and allowing wave components to interact according to Hasselmann's theory. Typically the sea is represented by 300 components: 25 wavelengths going in 12 directions (30°). Each component is allowed to propagate from grid point to grid point, growing with the wind or decaying in time, all the while interacting with other waves in the spectrum. To reduce computing time, the models use a nested grid of points: the grid has a high density of points in storms and near coasts and a low density in other regions. Typically, grid points in the open ocean are 3° apart.

Some recent experimental models take the wave-forecasting process one step further by assimilating altimeter and scatterometer observations of wind speed and wave-height into the model. Forecasts of waves using assimilated satellite data are available from the European Centre for Medium-Range Weather Forecasts. Details of the third-generation models produced by the Wave Analysis Group (WAM) are described in the book by Komen *et al*. (1994).

Measurement of Waves

Because waves influence so many processes and operations at sea, many techniques have been invented for measuring waves. Here are a few of the more commonly used. Stewart (1980) gives a more complete description of wave measurement techniques, including methods for measuring the directional distribution of waves.

Satellite Altimeters Satellite altimeters are now the most widely source of wave measurements. Altimeters were flown on Seasat in 1978, Geosat from 1985 to 1988, ERS-1 & 2 from 1991, Topex/Poseidon from 1992, Jason from 2001, and Envisat. Altimeter data are used to produce monthly mean maps of wave-heights and the variability of wave energy density in time and space. The data are also assimilated into wave forecasting models to increase the accuracy of wave forecasts.

The altimeter technique works as follows. Radio pulse from a satellite altimeter reflect first from the wave crests, later from the wave troughs. The reflection stretches the altimeter pulse in time, and the stretching is measured and used to calculate wave-height (Fig. 4). Accuracy is ±10%.

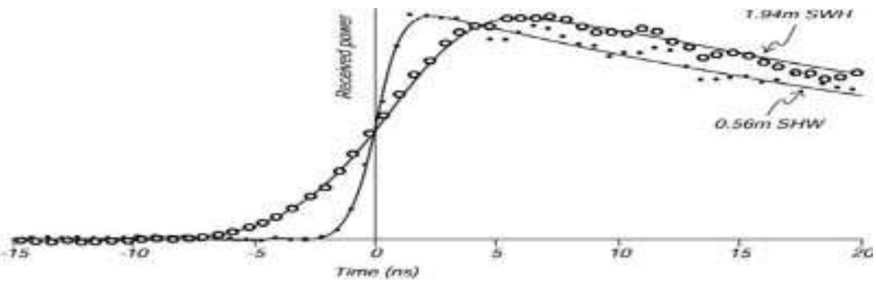

Fig. 4. Shape of radio pulse received by the Seasat altimeter, showing the influence of ocean waves. The shape of the pulse is used to calculate significant wave-height. (Adopted from Stewart, 1985).

Synthetic Aperture Radars on Satellites: These radars map the radar reflectivity of the sea surface with spatial resolution of 6-25 m. Maps of reflectivity often show wave-like features related to the real waves on the sea surface. I say "wave-like" because there is not an exact one-to-one relationship between wave-height and image density. Some waves are clearly mapped, others less so. The maps, however, can be used to obtain additional information about waves, especially the spatial distribution of wave directions in shallow water (Vesecky and Stewart, 1982). Because the directional information can be calculated directly from the radar data without the need to calculate an image (Hasselmann and Hasselmann, 1991), data from radars and altimeters on ERS–1 & 2 are being used to determine if the radar and altimeter observations can be used directly in wave forecast programs.

Accelerometer Mounted on Meteorological or Other Buoy: This is a less common measurement, although it is often used for measuring waves during short experiments at sea. The most accurate measurements are made using an accelerometer stabilized by a gyro so the axis of the accelerometer is always vertical.

Double integration of vertical acceleration gives displacement. In addition, the buoy's heave is not sensitive to wavelengths less than the buoy's diameter, and buoys measure only waves having wavelengths greater than the diameter of the buoy. Overall, careful measurements are accurate to ±10% or better.

Wave Gages: Gauges may be mounted on platforms or on the seafloor in shallow water. Many different types of sensors are used to measure the height of the wave or subsurface pressure which is related to wave-height. Sound, infrared beams and radio waves can be used to determine the distance from the sensor to the sea surface provided the sensor can be mounted on a stable platform that does not interfere with the waves. Arrays of bottom-mounted pressure gauges are useful for determining wave directions. Thus arrays are widely used just offshore of the surf zone to determine offshore wave directions.

Pressure gauge must be located within a quarter of a wavelength of the surface because wave-induced pressure fluctuations decrease exponentially with depth. Thus, both gauges and pressure sensors are restricted to shallow water or to large platforms on the continental shelf. Again, accuracy is ±10% or better.

For wave prediction in coastal waters it is necessary to consider the effects of shoaling, refraction and bottom friction. The computations involved for these shallow water effects are normally time consuming and laborious.
Numerical wave models require adequate wind inputs over the vast sea areas. Wind data over large areas cannot be acquired by resorting to conventional ship-board measurements or from synoptic weather charts. There is a need to adapt remote sensing methods for providing wind inputs at synoptic time intervals at close grid spacing over the ocean. Hence, satellite derived information on winds is very crucial for wave models.

Studies on wave energy interactions between sea and swell need more emphasis in future wave modelling.

The complex circulation features which normally prevail in coastal waters cause significant changes in shallow water wave characteristics. Hence, in shallow water wave modelling the role of wave-current interactions have to be incorporated.

CONCLUSION

Wavelength and frequency of waves are related through the dispersion relation. The velocity of a wave phase can differ from the velocity at which wave energy propagates. Waves in deep water are dispersive, longer wavelengths travel faster than shorter wavelengths. Waves in shallow water are not dispersive. The dispersion of ocean waves has been accurately measured, and observations of dispersed waves can be used to track distant storms. The shape of the sea surface results from a linear superposition of waves of all possible wavelengths or frequencies travelling in all possible directions. Wave energy is proportional to variance of surface displacement. Waves are generated by wind. Strong winds of long duration generate the largest waves. Observations by mariners on ships and by satellite altimeters have been used to make global maps of wave height. Wave gauges are used on platforms in shallow water and on the continental shelf to measure waves. Bottom-mounted pressure gauges are used to measure waves just offshore of beaches. And synthetic-aperture radars are used to obtain information about wave directions.

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