ISSN 1994-1978 (Web Version)

Journal of Soil and Nature (JSN)

(J. Soil Nature)

Volume: 8

Issue: 2

July 2015

J. Soil Nature 8(2): 11-17 (July 2015) DEVELOPING AN EQUATION FOR ESTIMATING INFILTRATION RATE THROUGH SOIL USING SCALING METHODS OF FURROW IRRIGATION A. TAVAKOLI, H. BABAZADEH, F. ABBASI AND H. SEDGHI



DEVELOPING AN EQUATION FOR ESTIMATING INFILTRATION RATE THROUGH SOIL USING SCALING METHODS OF FURROW IRRIGATION

A. TAVAKOLI¹, H. BABAZADEH², F. ABBASI³ AND H. SEDGHI⁴

¹Ph.D. Candidate, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ²Associate Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ³Professor, Agricultural Engineering Research Institute, Irrigation Dept., Karaj, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng. Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng., Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng., Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng., Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng., Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng., Dept., Science and Research Branch, Islamic Azad University, Tehran, Iran; ⁴Professor, Water Eng., Dept.,

*Corresponding author & address: Dr. Hossein Babazadeh, E-mail: h_babazadeh@srbiau.ac.ir Accepted for publication on 11 June 2015

ABSTRACT

Tavakoli A, Babazadeh H, Abbasi F, Sedghi H (2015) Developing an equation for estimating infiltration rate through soil using scaling methods of furrow irrigation. J. Soil Nature 8(2), 11-17.

Design, evaluation and simulation of surface irrigation systems depend on knowledge about soil infiltration characteristics and water movement in field. Management of surface irrigation and especially furrow irrigation is costly, time-consuming and complicated because of spatial and temporal variability of infiltration. Thus, infiltration parameters for different inflow discharges, furrow geometry and soil water content, are variable in furrow irrigation. Consequently, it is difficult to present a general equation for infiltration. Scaling is one of suitable methods to obtain a general relationship for infiltration. In this study, an appropriate equation achieved for scaling the infiltration components, using dimensional analysis and 12 distinctive furrow data. The superiorities of suggested equation over available equations in the literature are requiring less data and also easier calculation method. Required parameters comprising inflow discharge, flow depth in furrow, application time and advance time. Results demonstrated that applying scale factor caused dissimilar infiltration curves to merge and produce one curve. The values of R² and RMSE for suggested equation are 0.968 and 0.066, respectively.

Key words: furrow irrigation, dimensional analysis, scaling, infiltration

INTRODUCTION

Due to lower cost and consumed energy, surface irrigation is applied more than pressurized irrigation systems and is the predominant method of irrigation throughout the world (Vico and Porporato, 2011). Furrow irrigation is the most common type of surface irrigation and provides better on-farm water management capabilities under most surface irrigation conditions (Walker 2003). If accurate irrigation management is applied and spatial and temporal variations of soil properties are taken into consideration, achieving high efficiencies in surface irrigation will not impossible (Gillies et al. 2007). Modern techniques such as every-other-furrow irrigation result in less deep percolation and increase in water use efficiency (Sepaskhah and Kamgar-Haghighi, 1997). Infiltration of water into soil in surface irrigation is a complicated process that depends on several parameters including unsaturated hydraulic conductivity, soil roughness and furrow geometrical properties, which their in situ estimation if not difficult is time-consuming. On the other hand, the infiltration characteristics of soil are highly influential on surface irrigation performance (McClymont and Smith, 1996 and Oyonarte et al. 2002) and constitute the basic information required for designing an irrigation system (Machiwal et al. 2006). Spatial and temporal variation of infiltration rates makes the management of surface irrigation systems a very complex procedure (Rasoulzadeh and Sepaskhah, 2003 and Khatri and Smith, 2006). Infiltration parameters may vary for different discharges and initial soil moistures in furrow irrigation. Adjusting inflow discharge and cutoff time result in optimizing the performance of the irrigation relative to infiltration variations (Smith et al. 2005). Furthermore, inflow discharge affects infiltration through changing water depth and wetted perimeter (Enciso-Medina et al. 1998).

The importance of infiltration process resulted in development of a number of simple predicting analytical models. They are categorized into experimental models (Kostiakov 1932; Horton 1940 and Holtan 1961) and physical models (Green-Ampt 1911 and Philip 1957). Only a few of them have been successfully applied to field data (Machiwal *et al.* 2006). Parchami Araghi *et al.* (2010) reported that compared to other infiltration models (Kostiakov, Horton and Philip), the performance of Kostiakov-Lewis model was the best and the most suitable alternative to describe infiltration of water into the soil during furrow irrigation. Various studies used Kostiakov-Lewis equation for estimating infiltration and presented methods to calculate its parameters (Elliot and Walker, 1982; Elliot *et al.* 1983; Hopmans 1989 and Scaloppi *et al.* 1995).

Scaling is one of the ways to determine parameters of infiltration of water into the soil that is widely used widely nowadays. Scaling concept was first expressed by Miller and Miller (1956) based on dissimilar media theory. It has extensively been applied in order to specify spatial variations of soil hydraulic properties (Nielsen *et al.* 1998; Sposito 1998 and Warrick 1998). Youngs and Price (1981) scaled one-dimensional vertical downward infiltration for soils with different sizes and forms. Warrick *et al.* (1985) generalized semi-analytical solution of Philip's equation for one-dimensional infiltration using scaling method. Warrick and Hussen (1993) utilized scaling techniques to solve Richards' equation.

Recently, new methods have been proposed that decrease the data required for estimation of infiltration attributes. Employing dimensional analysis and scaling methods, Rasoulzadeh and Sepaskhah (2003) used eight measured infiltration equations for six soil series to obtain a generalized equation for infiltration. Resulted equation was a function of the wetted perimeter and the volume of applied water. Evaluation of the scaled

infiltration equation showed that it is applicable to other furrows in soils with different textures and hydraulic conditions and estimates the infiltration accurately. Khatri and Smith (2006) suggested a new method to estimate infiltration characteristics that used one advance point and a model infiltration curve. They formulated a scaling factor that was applied to scale the infiltration curves for the whole field, in conjunction with the Kostiakov-Lewis infiltration model. Applying this method in the field is simple and only requires one advance point in furrow, inflow discharge and wetted perimeter. Ahuja *et al.* (2007) scaled Kostiakov-Lewis infiltration model parameters based on the relations between them and the effective saturated hydraulic conductivity. Using scaling method, Sadeghi *et al.* (2008) developed a model to predict soil moisture profile in redistribution process that estimated its value in different depths and times. Karami *et al.* (2012) used scaling to quantify infiltration parameters and fitted resulted data to infiltration models (comprising of Kostiakov, Kostiakov-Lewis and Philip). These researchers reported that two-parameter Philip's equation with highest R^2 is the best model.

Literature reviews show that relations and models obtained for estimating infiltration till now are not accurate enough or require so much input data that their application is expensive and time-consuming (Rasoulzadeh and Sepaskhah, 2003 and Khatri and Smith, 2006). Considering the complicated process of water infiltration into soil in furrow irrigation and its dependency on several parameters including furrow length, application time, advance time, inflow discharge, furrow spacing, wetted perimeter, wetted area, initial soil moisture, water depth in furrow, unsaturated hydraulic conductivity, Manning's roughness coefficient and furrow slope, it is imperative to offer an equation with acceptable accuracy that would in the meantime require less data and simple calculation procedure. Thus, considering the mentioned features, a new equation was developed using scaling method to estimate infiltration in the current research.

MATERIALS AND METHODS

Data used in this study were extracted from 12 tests of furrow irrigation with regulated outflow in a corn field with 15 irrigation events in each test, which were conducted over three consecutive years (2008 and 2010) in seed and plant improvement research institute, Karaj, Iran (35.56° N, 50.58° E and 1312 m a.s.l.) (Abbasi and Chogan, 2011). Longitudinal slope, furrow length and furrow spacing were 0.006 m/m, 165 m and 0.75 m, respectively. Soil texture determined by hydrometric method that was loam for all tested furrows. In order to minimize runoff, the cutback regime applied during all irrigations. Inflow and outflow discharges were measured by using a volumetric counter and WSC flume (type 3), respectively. The mean values for some properties of tested furrows are provided in Table 1. To draw the cumulative infiltration vs. time, inflow and outflow discharge were measured in different times after the beginning of irrigation. Then, employing two-point method of Elliott and Walker (1982) and water balance procedure described by Walker and Skogerboe (1987), coefficients of Kostiakov-Lewis equation comprising *a*, *k* and *f*₀ were estimated. Next, the cumulative infiltrations were plotted against time for the investigated furrows (Fig. 1).

Furrow/Characteristics	Inflow (l s ⁻¹)	Application Time (min)	Advance Time (min)
Test 1	0.2594	602	47
Test 2	0.353	348	43
Test 3	0.371	268	46
Test 4	0.283	580	64
Test 5	0.294	335	71
Test 6	0.317	261	104
Test 7	0.352	604	36
Test 8	0.361	302	37
Test 9	0.349	239	92
Test 10	0.362	605	40
Test 11	0.318	488	34
Test 12	0.365	368	34

Table 1. Averages for some characteristics of tested furrows

In this study, the infiltration process was scaled by using dimensional analysis based on Buckingham's (1914) Π theory. Dimensional analysis is a mathematical method to establish relations between quantified physical variables using field or laboratory experiments. It converts physical variables to dimensionless quantities which results in reducing the required tests. Dimensional analysis is an effective tool for modeling problems that cannot be solved analytically and need iterative tests (Massey and Ward-Smith, 2006).

The essential part of a scaling method is to find the characteristic time (T_c) and space scales (L_c) for the system to be modeled. To formulate the L_c and T_c , effective parameters must be considered. Effective parameters for T_c and L_c can be the same as those effective in infiltration. Infiltration into furrows depends on many parameters. It is evident that considering all of the effective parameters may cause intricacy of the conditions. Therefore, in this study, effects of different parameters were evaluated and important parameters were selected for considered in the dimensional analysis.

Dimensional analysis

A useful tool in modern fluid mechanics, which is closely related to the laws of similitude, is dimensional analysis. One of the chief advantages of employing dimensional analysis in an experimental investigation is that it often permits an investigator to obtain experimental results with a minimum amount of labor and maximum ease of application. In this study, Buckingham's Π theorem (1914) is used. According to previous investigations, the variables that affect infiltration and also the L_c and T_c can be written as follows:

$$L_{c} = f (L, W, Q, t_{c}, t_{p}, D, H, S_{0}, Z)$$
(1)

$$T_{c} = f (L, W, Q, t_{c}, t_{p}, D, Z)$$
(2)

where L_c is the scaling factor (m⁻²), f denotes an unknown function, L is furrow length (m), W represents furrows spacing (m), Q is inflow discharge measured at t_c (m³ min⁻¹), t_c represents a known time after the beginning of irrigation which determined by the user (min), t_p denotes time for water to advance to the end of the furrow (min), D is initial soil moisture (m³ m⁻³), H represents water depth in furrow at t_c (m), S₀ denotes furrow slope (m m⁻¹), Z is cumulative infiltration (m³ m⁻¹), and T_c represents the scaling factor (min⁻¹), Using 80% of the input data, a series of approximations were tested and after several trial and error procedures and from more than 40 diverse Buckingham's II, calculated for each feature, eventually, the suitable equations for L_c and T_c were obtained, and the remaining data were used for model validation. Using Minitab statistical software, the best fitted equation was derived for Figure 2.

Statistical criteria employed to evaluate the suggested model were R^2 , root mean square error (RMSE) (Eq. 3), and mean bias error (MBE), (Eq. 4). Providing term-by-term comparison of the difference between predicted and measured values where the RMSE value represents information regarding short-term model performance. The MBE was the index for evaluating under- or overestimation of the model. The proposed model was assessed based on minimum values of MBE (absolute value) and RMSE and maximum value of R^2 .

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (Z_{actual} - Z_{predicted})^{2}}{n}}$$

$$MBE = \frac{\sum_{i=1}^{n} (Z_{actual} - Z_{predicted})}{n}$$
(3)
(4)

where Z_{actual} represents measured infiltration, Z_{scaled} represents scaled infiltration and n is the number of data.

RESULTS AND DISCUSSION

The accumulated infiltrations of different tested furrows were plotted against time (Fig. 1). Different values of accumulated infiltration for various tests are due to diverse physical soil properties (including soil structure, initial soil moisture, unsaturated hydraulic conductivity, and roughness) and hydraulic properties of furrows (such as wetted perimeter and area).

. .



Fig. 1. Accumulated infiltration vs. time for all used data

Using 80% of input data, final equations for scaled time (T^*) and scaled infiltration (Z^*) were calculated as follows:

$$Z^* = L_c \times Z = \frac{H \times t_p}{Q \times t_c^2} \times Z$$

$$T^* = T_c \times t_c = \frac{1}{Q} \times t_c$$
(5)
(6)

where Q denotes inflow discharge (m³ min⁻¹), t_p is advance time (min), H represents water depth in furrow (m), Z is accumulated infiltration (m³ m⁻¹) and t_c denotes application time (min). Then, T^{*} was plotted against Z^{*} (Fig. 2).

Tavakoli et al.



Fig. 2. Scaled accumulated infiltration vs. scaled time for 80% of used data

Next, using statistical software Minitab 17, the best fitted line for 80% of scaled data was plotted (Fig. 2) and eq. (7) was obtained. Gauss-Newton algorithm was used in Minitab to estimate the following equation.

$$Z^* = 0.00445 \times \left(T^*\right)^{-1.0064} \tag{7}$$

Statistical parameters of eq. (7) are presented in Table 2. Positive value of MBE implies overestimation of infiltration calculated from eq. (7) compared to measured infiltration.

Table 2. Statistical parameters of suggested equation

\mathbf{R}^2	RMSE	MBE
0.968	0.066	-0.044
DMCE		MDE

RMSE: root mean square error, MBE: mean bias error

In a similar study, using surface tension, water density, viscosity of water, change in soil water content, saturated hydraulic conductivity and wetted perimeter as inputs, Rasoulzadeh and Sepaskhah (2003) derived the following scaled infiltration equation:

$$Z^* = 0.1283 \times \left(T^*\right)^{0.3301} + 2.3161 \times T^*$$
(8)

Rasoulzadeh and Sepaskhah (2003) only reported R^2 value for proposed equation (0.9976). Although there is not similarity between eqs. (7) and (8), the suggested equation in the current study requires input data which are much more readily-obtainable (inflow discharge, advance time, application time and water depth in furrow) and make it more practical.

Aiming at reducing input data for infiltration, Khatri and Smith (2006) scaled the cumulative infiltration and reported $R^2 = 0.9259$ for the actual cumulative infiltration at a particular advance time (200 min) for each of the 27 irrigation events at field against the scaled cumulative infiltration for the same events, and $R^2 = 0.9973$ for advance times ranging from 50 to 600 min. Based on R^2 values, it could be concluded that the result of Khatri and Smith (2006) is slightly better than our result. The required data were input data for two-point method proposed by Elliot and Walker (1982) and McClymont and Smith (1996) method, the estimation of which is more time-consuming and expensive than that needed for input data of eq. (7) and could also insert errors in calculation.

Evaluating EVALUE, SIPAR_ID, and INFILT models for the estimation of furrow irrigation infiltration, using 12 data sets corresponding to blocked-end and free-draining furrows, Etedali *et al.* (2011) found RMSE and R^2 values as 1.14 and 0.54, respectively for the EVALUE, and 1.139 and 0.87 for SIPAR_ID. Comparing the results of RMSE and R^2 for our equation (0.066 and 0.968, respectively) with the results of Etedali *et al.* (2011), suggested the higher accuracy of the obtained equation in the current study.

Scaled and measured accumulated infiltration for 20% of data was used to validate the suggested equation. Results of four tests, as examples out of total tests, are provided in Fig. 3.



Fig. 3. Comparing scaled and measured accumulated infiltration (Test 6-9)

As shown in Fig. 3, the suggested equation only slightly underestimated the measured accumulated infiltration values. However, close correlation between fitted line and 45 degree line indicated sufficient prediction accuracy. Statistical parameters of these 12 tests are provided in Table 3.

Table 3. Statistical parameters of selected tests

Test No.	\mathbf{R}^2	RMSE	MBE
1	0.90	0.089	-0.052
2	0.97	0.068	-0.051
3	0.97	0.049	-0.036
4	0.99	0.081	-0.056
5	0.97	0.055	-0.035
6	0.98	0.028	-0.018
7	0.98	0.034	-0.014
8	0.96	0.022	-0.004
9	0.99	0.025	-0.019
10	0.99	0.122	-0.092
11	0.95	0.116	-0.078
12	0.97	0.104	-0.074

RMSE: root mean square error, MBE: mean bias error

Negative values of MBE in selected tests implied underestimation of scaled infiltration compared to measured values. Generally, eq. (7) showed a slight underestimation (Table 2). Close values of R^2 to unity (>0.96) along with minor values of RMSE (<0.06) illustrate high precision of eq. (7). The accuracy of suggested equation is

comparable to the results of Rasoulzadeh and Sepaskhah (2003) and Khatri and Smith (2006), although its calculation is easier, faster and requires less input data. As can be seen from Table 3, for all test but the third one the statistical values are R^2 >0.97 and RMSE<0.08, which shows appropriate accuracy of suggested equation.

CONCLUSION

Infiltration parameters are diverse for different inflow discharges, furrow geometry and soil water contents in surface irrigation. Consequently, it is difficult to present a general equation for infiltration. In this study, an appropriate equation was achieved for scaling the infiltration components, using dimensional analysis and twelve distinctive furrow data. The advantages of suggested equation over available equations in the literature are requiring less data and also easier calculation method. Required parameters are inflow discharge, flow depth in furrow, application time and advance time. Results demonstrated that applying scale factor caused dissimilar infiltration curves to merge and produce one curve. Furthermore, results of evaluating obtained scale factor demonstrated the potential of applying this equation in various furrows with various discharges. The research result is highly recommended that similar studies in furrows with different soil textures, lengths and slopes be conducted to obtain a general scaled equation that would require less data with appropriate accuracy.

REFERENCES

Abbasi F, Chogan R (2011) Investigation of surface fertigation on water productivity, yield, and yield components of corn in Karaj. Final Research Report, No. 40212. Agricultural Engineering Research Institute. 44 pp.

Ahuja LR, Kozak JA, Andales AA, Ma L (2007) Scaling parameters of the Lewis-Kostiakov water infiltration equation across soil textural classes and extension to rain infiltration. *J.T. ASABE*. 50(5), 1525-1541.

Buckingham E (1914) On physically similar systems; illustrations of the use of dimensional equations. J. Physical Review 4(4), 345–376

Elliot RL, Walker WR (1982) Field evaluation of furrow infiltration and advance functions. Trans. *J.ASAE*, 25, 396–400.

Elliot RL, Walker WR, Skogerboe GV (1983) Infiltration parameters from furrow irrigation advance data. J. *Transactions of the ASAE*, 26, 1726–1731.

Enciso-Medina J, Martin D, Einsenhaur D (1998) Infiltration model for furrow irrigation. J. Irrig. Drain. Eng. 124(2), 73–80.

Etedali HR, Ebrahimian H, Abbasi F, Liaghat A (2011) Evaluating models for the estimation of furrow irrigation infiltration and roughness. *Spanish Journal of Agricultural Research*, 9(2), 641-649.

Gillies M, Smith R, Raine R (2007) Infiltration parameters from surface irrigation advance and run-off data. J. Irrigation Science. 24(1), 25-35.

Green WH, Ampt GA (1911) Studies of soil physics—part I: the flow of air and water through soils. *Journal of Agricultural Sciences*, 4(1), 1–24.

Holtan HN (1961) A concept for infiltration estimates in watershed engineering. USDA-ARS, pp. 41–51.

Hopmans JW (1989) Stochastic description of field measured infiltration data. J. Transactions of the ASAE, 32, 1987–1993.

Horton RE (1940) An approach towards a physical interpretation of infiltration capacity. J. Soil Science Society of America Proceedings, 5, 399–417.

Karami A, Homaee M, Bybordi M, Mahmoodian Shoushtari M (2012) Quantifying parameters of infiltration of water into the soil using scaling. *Iran Water Research Journal*. 11(2), 65-73 (In Persian).

Khatri L, Smith R (2006) Real- time prediction of soil infiltration characteristics for the management of furrow irrigation. *J. Irrigation Science*. 25(1), 33-43.

Kostiakov AN (1932) On the dynamics of the coefficient of water percolation in soils and on the necessity of studying it from a dynamic point of view for the purposes of amelioration. Transactions of 6th Congress if International Society of Soil Science. Moscow. Russian. 17-21.

Machiwal D, Madan KJ, Mal BC (2006) Modelling infiltration and quantifying spatial soil variability in a wasteland of Kharagpur, India. *J. Biosyst. Eng.*, 95(4), 569-582.

Massey B, Ward-Smith AJ (2006) Mechanics of Fluids. 8th ed., Taylor and Francis, London.

McClymont DJ, Smith RJ (1996) Infiltration parameters from the optimization on furrow irrigation advance data. J. Irrig. Sci., 17, 15–22.

Miller EE, Miller RD (1956) Physical theory for capillary flow phenomena. *Journal of Applied Physics*, 27, 324–332.

Nielsen DR, Hopmans JW, Reichardt K (1998) An emerging technology for scaling field soil water behavior. In: Scale Dependence and Scale Invariance in Hydrology (Sposito G,ed), pp. 136–166. Cambridge University Press, Cambridge.

Oyonarte NA, Mateos L, Palomo MJ (2002) Infiltration variability in furrow irrigation. J. Irrig. Drain. Eng. 128(1), 26–33.

Parchami Araghi F, Mirlatifi SM, Ghorbani Dashtaki S, Mahdian MH (2010) Evaluating some infiltration models under different soil texture classes and land uses. *Iranian Journal of Irrigation and Drainage*. 2(4), 193-205.

Philip JR (1957) The theory of infiltration-3: moisture profiles and relation to experiment. J. Soil Sci. 84(2), 163-178.

Rasoulzadeh A, Sepaskhah AR (2003) Scaled infiltration equations for furrow irrigation. J. Biosyst. Eng. 86(3), 375-383.

Sadeghi M, Ghahreman B, Davary K (2008) Scaling and prediction of soil moisture profile during redistribution phase. *Journal of Water and Soil*. 22(2), 417-431 (In Persian).

Scaloppi EJ, Merkley GP, Willardson LS (1995) Intake parameters from advance and wetting phases of surface irrigation. *Journal of Irrigation and Drainage Engineering*, 121, 57–69.

Sepaskhah AR, Kamgar-Haghighi AA (1997) Water use and yield of sugarbeet grown under every-other furrow irrigation with different irrigation intervals. J. Agricultural Water Management, 34, 71–80.

Smith RJ, Raine SR, Minkevich J (2005) Irrigation application efficiency under surface irrigated cotton. J. Agricultural Water Management. 71, 117–130.

Sposito G (1998) Scale Dependence and Scale Invariance in Hydrology. Cambridge University Press, Cambridge, UK.

Vico G, Porporato A (2011) From rainfed agriculture to stress avoidance irrigation: I. A generalized irrigation scheme with stochastic soil moisture. J. Adv. Water Resour., 34(2), 263–271.

Walker WR (2003) SIRMOD III-Surface Irrigation Simulation, Evaluation and Design, Guide and Technical Documentation. Utah State University. pp. 146.

Walker WR, Skogerboe GV (1987) Surface Irrigation: Theory and Practice. Prentice-Hall, Englewood Cliffs, New Jersey. 386p.

Warrick AW (1998) Spatial variability. In: Environmental Soil Physics (Hillel D, ed), pp Academic Press, San Diego, CA. 655-676.

Warrick AW, Hussen AA (1993) Scaling of Richards equation for infiltration and drainage. J. Soil Sci. Soc. Am. J., 57, 15–18.

Warrick AW, Lomen DO, Yates SR (1985) A generalized solution to infiltration. J. Soil Sci. Soc. Am. J., 49, 34–38.

Youngs EG, Price RI (1981) Scaling of infiltration behavior in dissimilar porous materials. J. Water Resour. Res., 17, 1065–1070.